Lesson 16: Basic Control Modes

ET 438a Automatic Control Systems Technology

Learning Objectives

After this presentation you will be able to:

- Describe the common control modes used in analog control systems
- List the characteristics of common control modes
- Write the time, Laplace and transfer functions of common control modes
- Identify the Bode plots of common control modes
- Design OP AMP circuits that realize theoretical control mode performance.

Control Modes-Proportional Control Action

Process characteristics for optimum results:

1) Small process capacitance
2) Rapid load changes

Limitations: Small steady-state error may require high gain to achieve acceptable error levels

Mathematical representations

Time function: $v = K_p \cdot e + v_o$

Where: e = time domain error signal
K_p = proportional gain
v_o = controller output with e=0
v = controller time domain output

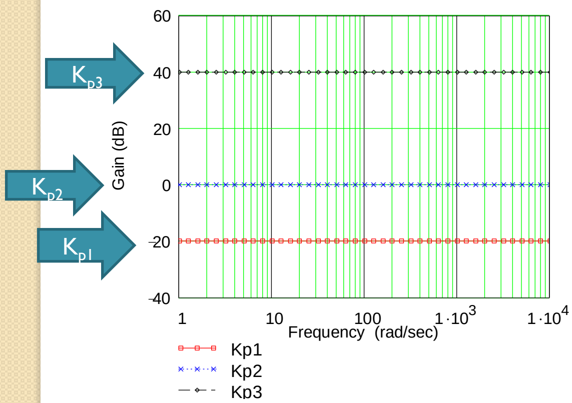
Laplace function: $V(s) = K_p \cdot E(s)$

Note: Initial condition v_o=0 on Laplace function

Transfer function: $\frac{V(s)}{E(s)} = K_p$

Proportional Control Frequency Response

Bode plots of three values of K_p: K_{p1}=0.1, K_{p2}=1, and K_{p3}=100



Note: gain is independent of frequency.

Practical realization:

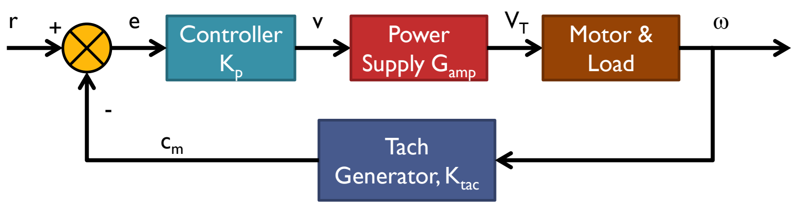
Non-inverting OP AMP circuit

Motor Speed Control

Example 16-1: Determine the effect of applying proportional control to the block diagram shown below. The motor produces the following results with the control loop open:

T_{L1} = 0.05 N-m
V_T = 19.24 Vdc
I_{a1} = 1.033 A
ω_1 = 300 rad/sec

T_{L2} = 0.075 N-m (50% increase in load)
V_T=19.24
I_{a2} = 1.45 A
ω_2 = 291.7 rad/sec



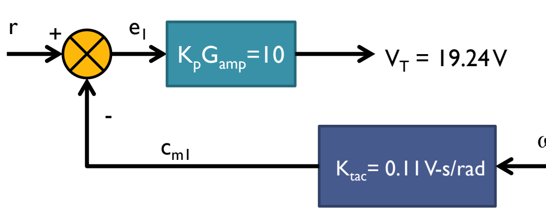
Motor Speed Control

Motor Parameters:

T_f = 0.012 N-m
K_T = 0.06 N-m/A
K_e = 0.06 V-sec/rad

R_a = 1.2 ohms
K_{tac} = 0.11 V-sec/rad
K_pG_{amp} = 10 V/V

Solution: Find the error produced and the setpoint value, r. Then write equations around control loop.



$$e_1 = r - c_{m1}$$

$$e_1 = \frac{V_T}{K_p G_{amp}}$$

ω =300 rad/sec

$$c_{m1} = K_{tac} \cdot \omega = (0.11\ \text{V-s/rad}) \cdot (300\ \text{rad/s}) = 33\ \text{V}$$

Example 16-1 Solution (2)

Combine equations

$$e_1 = r - c_{m1}$$

$$e_1 = \frac{V_T}{K_p G_{amp}}$$

$$\frac{V_T}{K_p G_{amp}} = r - c_{m1} \Rightarrow \frac{V_T}{K_p G_{amp}} + c_{m1} = r$$

Substitute values $\frac{19.24\text{ V}}{10\text{V/V}} + 33\text{ V} = 34.924\text{ V}$

Simplify $r = 34.924\text{ V}$

Compute error $e_1 = r - c_{m1}$

Substitute values $e_1 = 34.924 - 33.0\text{ V}$

Simplify $e_1 = 1.924\text{ V}$

The initial setpoint value of r=34.924V with an error of 1.924V at a speed of 300 rad/s and T_{L1}=0.05 N-m

Example 16-1 Solution (3)

When torque increase to T_{L2}=0.075 N-m new V_T is defined by…

$$V_{T2} = (r - c_{m2}) \cdot K_p \cdot G_{amp}$$

Substitute in Tachometer formula

$$V_{T2} = (r - K_{tac} \cdot \omega_2) \cdot K_p \cdot G_{amp}$$

Since setpoint, r does not change error must change due to measured speed change.

Motor equations

$$V_{T2} = I_{a2} \cdot R_a + e_{b2}$$

$$e_b = K_e \cdot \omega_m$$

Combine these equations to get:

$$V_{T2} = I_{a2} \cdot R_a + K_e \cdot \omega_2$$

Need two equations to find both V_{T2} and ω_{m2}.

Example 16-1 Solution (4)

Substitute in know values and simplify equation to get first relationship.

r_1=34.924 V K_pG_{amp}=10 K_{tac} = 0.11 V-s/rad

$$V_{T2} = (r - K_{tac} \cdot \omega_2) \cdot K_p \cdot G_{amp}$$
$$V_{T2} = (34.924 - 0.11 \cdot \omega_2) \cdot 10$$
$$V_{T2} = 349.24 - 1.1 \cdot \omega_2 \quad (1)$$

Equation 1

Now use the motor armature circuit equation and the armature current for T_{L2} =0.075 N-m to find second independent equation.

I_{a2} =1.45 A K_e = 0.06 V-s/rad R+ =1.2 ohms

$$V_{T2} = I_{a2} \cdot R_a + K_e \cdot \omega_2$$
$$V_{T2} = (1.45\ \text{A}) \cdot (1.2\ \Omega) + (0.06\ \text{V-s/rad}) \cdot \omega_2$$
$$V_{T2} = 1.74 + 0.06 \cdot \omega_2 \quad (2)$$

Equation 2

Example 16-1 Solution (5)

Place equations (1) and 2) into standard form and solve simultaneously using software or calculator.

$$V_{T2} = 349.24 - 1.1 \cdot \omega_2$$
$$V_{T2} + 1.1 \cdot \omega_2 = 349.24 \quad (1)$$
$$V_{T2} = 1.74 + 0.06 \cdot \omega_2$$
$$V_{T2} - 0.06 \cdot \omega_2 = 1.74 \quad (2)$$

$$V_{T2} = 19.71\ \text{V} \qquad \omega_2 = 299.6\ \text{rad/sec}$$

Answers

Now compute the error signal from the new tachometer output voltage c_{m2}.

$$c_{m2} = K_{tac} \cdot \omega_2$$
$$c_{m2} = (0.11 \cdot \text{V} - \text{s/rad}) \cdot (299.6\ \text{rad/sec}) = 32.956\ \text{V}$$
$$e_2 = (r - c_{m2}) = 34.924 - 32.956 = 1.968\ \text{V}$$

Error increases $e_2 > e_1$ 1.968>1.924 V to rebalance system

Example 16-1 Solution (6)

Now determine the percentage speed changes for open loop and feedback control. Setpoint, r=300 rad/sec

Open loop speed change

$$\left[\frac{\omega_r - \omega_l}{\omega_r}\right]\cdot 100\% = \%SE \quad (SE = \text{speed error})$$

$$\left[\frac{300 - 291.7}{300}\right]\cdot 100\% = \%SE$$

$$2.77\% = \%SE$$

Answers

Feedback loop speed change

$$\left[\frac{300 - 299.6}{300}\right]\cdot 100\% = \%SE$$

$$0.143\% = \%SE$$

Answers

Feedback reduces speed error by factor of 19.35

Integral Control Mode

Integral Mode characteristics:

1) Output is integral of error over time
2) Drives steady-state error to zero
3) Adds pole to transfer function at s=0 (infinite gain to constant)
4) Integrators tend to make systems less stable

Equations

Time: $v(t) = K_I \cdot \int_0^{\tau} e(t)\,dt + v_0$

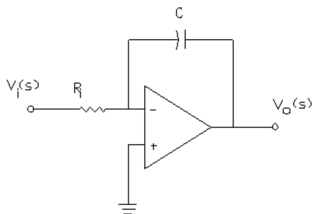
Where K_I = integral gain constant

Laplace: $V(s) = K_I \cdot \left[\frac{1}{s}\right] \cdot E(s$

Transfer Function: $\frac{V(s)}{E(s)} = K_I \cdot \left[\frac{1}{s}\right]$

OP AMP Realizations of Integral Control

Ideal OP AMP Integrator

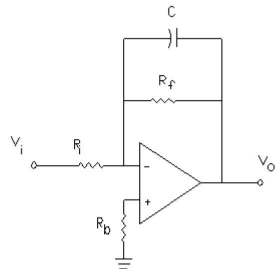


Transfer Function

$$\frac{V_o(s)}{V_i(s)} = \frac{-1}{R_i \cdot C} \cdot \left[\frac{1}{s}\right] \qquad K_I = \frac{-1}{R_i \cdot C}$$

One pole at s=0

Practical OP AMP Integrator



Transfer Function

$$\frac{V_o(s)}{V_i(s)} = \left[\frac{-R_f}{R_i}\right]\left[\frac{1}{1+R_f \cdot C \cdot s}\right]$$

One pole at $s=-1/R_f \cdot C$

Bode Plots of Integrator Circuits

Substitute $j\omega$ for s and find the magnitude and phase shift of the transfer function for different values of ω.

Ideal Integrator

$$G(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-1}{R_i \cdot C \cdot j\omega}$$

Practical Integrator

$$G(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \left[\frac{-R_f}{R_i}\right]\left[\frac{1}{1+R_f \cdot C \cdot j\omega}\right]$$

Take magnitude and phase shift of each of these functions using rules of complex numbers.

z=a+jb

Magnitude of z: z=|z|

$$|z| = \sqrt{a^2+b^2} = \sqrt{Re(z)^2 + Im(z)^2}$$

Scale for dB $\quad dB = 20 \cdot \log(|G(j\omega)|)$

Phase Shift

$$\phi = \tan^{-1}\left[\frac{b}{a}\right] = \tan^{-1}\left[\frac{Im(z)}{Re(z)}\right]$$

Bode Plots of Integrator Circuits

Practical Integrator Circuit

$$G(j\omega)=\left[\frac{-R_f}{R_i}\right]\left[\frac{1}{1+R_f\cdot C\cdot j\omega}\right]$$

Taking magnitude gives

$$|G(j\omega)|=\left[\frac{-R_f}{R_i}\right]\left[\frac{1}{\sqrt{1+R_f^{\ 2}\cdot C^2\cdot\omega^2}}\right]$$

$$dB=20\cdot\log(|G(j\omega)|)$$

Phase Shift gives

$$\phi(\omega)=180-\tan^{-1}[R_f\cdot C\cdot\omega]$$

180 degree phase shift is from inverting configurations

Ideal Integrator Circuit

Magnitude gives

$$|G(j\omega)|=\frac{1}{R_i\cdot C}\qquad dB=20\cdot\log(|G(j\omega)|)$$

Phase shift $\frac{1}{j}=-j=-90^0$

$$\phi=180^\circ-90^\circ=90^\circ\quad\text{Contant value}$$

Integrator Bode Plots Using MatLAB

$$G(j\omega)=\left[\frac{-R_f}{R_i}\right]\left[\frac{1}{1+R_f\cdot C\cdot j\omega}\right]\qquad G(j\omega)=\frac{-1}{R_i\cdot C\cdot j\omega}$$

Use MatLAB script to generate Bode plots and transfer function.

Define parameters: R_i = 10 kΩ, R_f = 100 kΩ, C = 0.01 μF

MatLAB Script

```
ri=input('Enter value of input resistance: ');
c=input('Enter value of capacitance: ');
rf=input('Enter value of feedback resistance: ');

% compute transfer function model parameters for
% practical integrator

tau=rf*c;
ki=-rf./ri;
```

Input statement

Comments begin with %

Integrator Bode Plots Using MatLAB

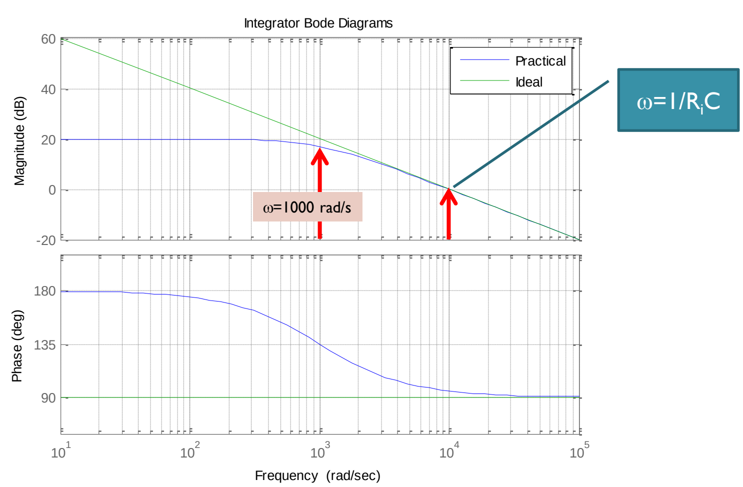
MatLAB Script (Continued)

```
% compute parameter for ideal integrator
tau1 = ri*c;

% build transfer functions
% denominator form is a1*s^2+a2s+a3

Av=tf([ki],[tau 1])
Av1=tf([-1],[tau1 0])

%plot both on the same graphs
bode(Av,Av1);
```

Create transfer functions

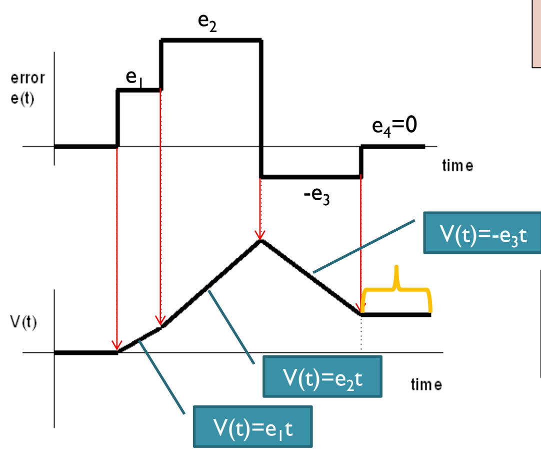
Plot both graphs on same figure

Integrator Bode Plots Using MatLAB



Integral Action on Time Varying Error Signals

Integral of constant, k, is line with slope k.



Integrator produces a linearly increasing output for constant error input

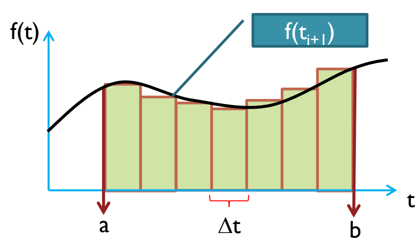
Negative error causes decreasing output

Zero error maintains last output value

Estimating Integrator Output

From Calculus, integral is sum of area below a function plot

$$\int_a^b f(t)\,dt \approx \sum_{i=0}^{n} \left(f(t_{i+1})\right)\cdot \Delta t \qquad \text{Where } \Delta t = t_{i+1} - t_i = \frac{b-a}{n}$$



For linear error plots, integral is the sum of the areas of linear segments.
Use triangle, trapezoid, and rectangle formulas to approximate output

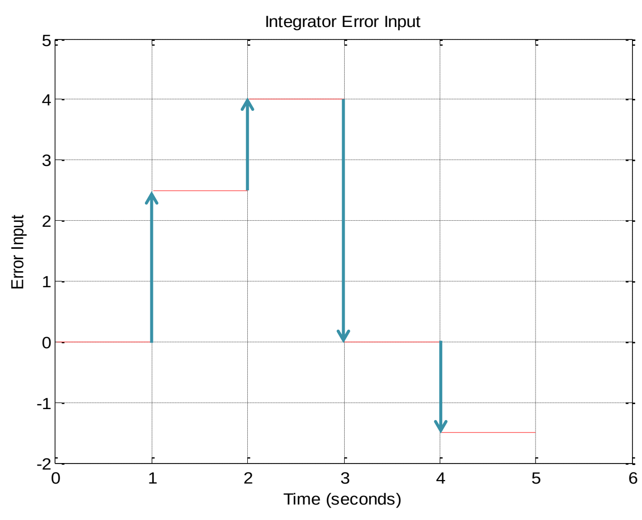
Integrator Output

Example 16-2: An ideal integrator has a gain of K_I =0.1 V/s. Its initial output is v=1.5 V. Determine the integrator outputs if the error has step increases given by the table below.

Error Magnitude (V)	Time Interval
e(t)=0	$0 \leq t \leq 1$ seconds
e(t)=2.5	$1 < t \leq 2$ seconds
e(t)=4	$2 < t \leq 3$ seconds
e(t)=0	$3 < t \leq 4$ seconds
e(t)=-1.5	$4 < t \leq 5$ seconds

Example 16-2 Solution (1)

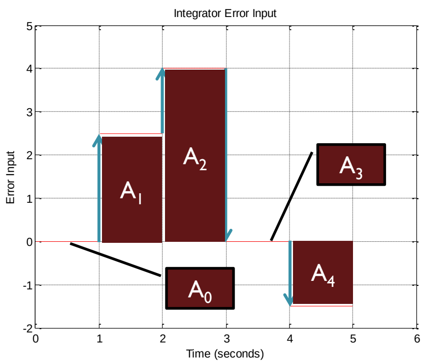
Plot the error function that is input to the integrator



Example 16-2 Solution (1)

Use the approximate formula to find the error at the end of each interval

$$v_i = K_I \cdot \sum_{i=0}^{n} e_i \cdot \Delta t \qquad \text{Where } \Delta t = 1 \ \ n = 5$$



$V_0 = 1.5$

$A_0 = K_I \cdot \Delta t \cdot e_0 = 0.1(1)(0) = 0$

$V_1 = 1.5 + A_0 = 1.5 + 0 = 1.5$

$A_1 = K_I \cdot \Delta t \cdot e_1 = 0.1(1)(2.5) = 0.25$

$V_2 = 1.5 + A_1 = 1.5 + 0.25 = 1.75$

$A_2 = K_I \cdot \Delta t \cdot e_2 = 0.1(1)(4) = 0.40$

$V_3 = 1.75 + A_2 = 1.75 + 0.40 = 2.15$

$A_3 = K_I \cdot \Delta t \cdot e_3 = 0.1(1)(0) = 0$

$V_4 = 2.15 + A_3 = 2.15 + 0.0 = 2.15$

$A_4 = K_I \cdot \Delta t \cdot e_4 = 0.1(1)(-1.5) = -0.15$

$V_5 = 2.15 + A_4 = 2.15 + -0.15 = 2.00$

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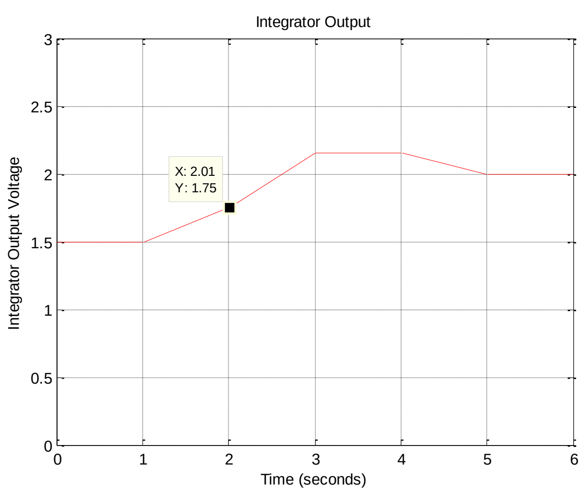
Example 16-2 Solution (2)

Integrator output plot



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Derivative Control Mode

Derivative Control Characteristics:

1) Produces output only when error is changing
2) Output is proportional to rate of change in error
3) Derivative control never used alone
4) Used with proportional and/or integral modes
5) Anticipates error by observing the rate of change

Derivative Mode Equations

Time Equation: $v(t) = K_d \cdot \frac{de(t)}{dt}$

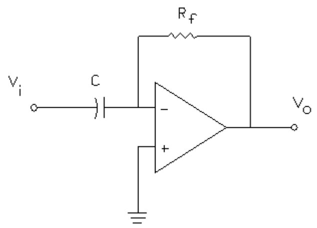
Laplace Equation: $V(s) = K_d \cdot s \cdot E(s)$

Transfer Function Equation: $\frac{V(s)}{E(s)} = K_d \cdot s$

Differentiators are high-pass filters to sinusoidal signals. They increase sensitivity to rapid error changes when added to controllers.

OP AMP Realizations of Differentiators

Ideal OP AMP Differentiator

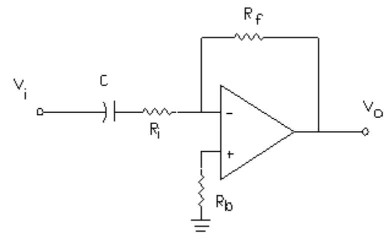


Transfer Function

$$\frac{V_o(s)}{V_i(s)} = -R_i \cdot C \cdot s$$

Introduces one zero at s=0

Practical OP AMP Differentiator



Transfer Function

$$\frac{V_o(s)}{V_i(s)} = \frac{-R_f \cdot C \cdot s}{1 + R_i \cdot C \cdot s}$$

Introduces: zero at s=0

pole at $s=-1/R_i \cdot C$

Bode Plots of Differentiators

Ideal Differentiator Equations

$$G(j\omega) = \frac{V_o(s)}{V_i(s)} = -R_f \cdot C \cdot j\omega$$

$$|G(j\omega)| = R_f \cdot C \cdot \omega$$

$$dB = 20 \cdot \log[|G(j\omega)|]$$

$$\phi = 90^\circ \quad \text{Constant over all } \omega$$

Practical Differentiator Equations

$$G(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-R_f \cdot C \cdot j\omega}{1 + R_i \cdot C \cdot j\omega}$$

$$|G(j\omega)| = \frac{R_f \cdot C \cdot \omega}{\sqrt{1 + R_i^2 \cdot C^2 \cdot \omega^2}}$$

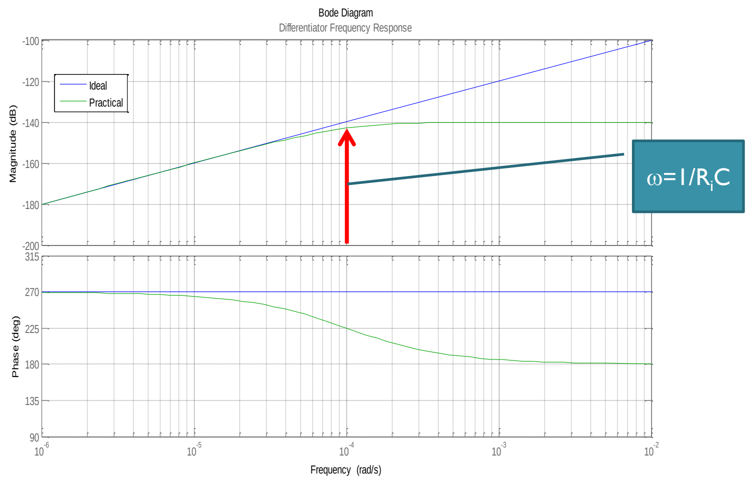
$$dB = 20 \cdot \log[|G(j\omega)|]$$

$$\phi = 270^\circ - \tan^{-1}(R_i \cdot C \cdot \omega)$$

Use MatLAB script to generate Bode plots and transfer function.

Define parameters: R_i = 10 kΩ, R_f = 100 kΩ, C = 0.01 μF

Bode Plots of Differentiators



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END LESSON 16: BASIC CONTROL MODES